Final Exam Electricity and Magnetism 2, 7-Nov-2016

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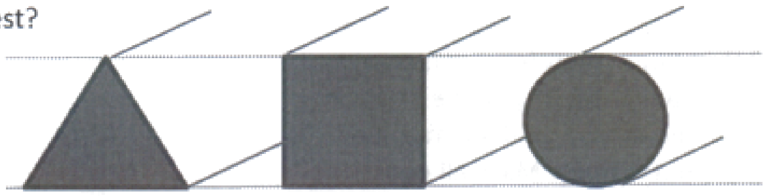
You do not need to use separate sheets for the individual questions. Please write your name and student number on each sheet. Number all answers clearly, especially if they are out of order. If you need the result of an earlier part of a question but do not know the answer, make a logical assumption and indicate this in your answer.
There are 7 questions with a total of 90 points on the test.

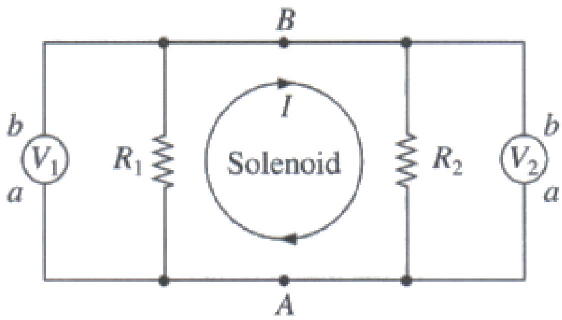
1. Quick Questions [15 points]
   a. [4 points] Three wires with different shaped cross sections are made from the same material with conductivity σ. Which wire has the highest resistance and which the lowest?



   b. [6 points] A traveling EM wave has $E_z = E_0 \cos(ky - \omega t)$, $E_x = E_0 \sin(ky - \omega t)$ and $E_y = 0$
      I. What is the direction of propagation of this wave?
      II. What is the polarization of this wave?

   c. [5 points] Which of the following are traveling waves. Indicate the direction of motion and the magnitude of the velocity
      I. $E(x,t) = E_0 e^{ik(x-y)} e^{-i\omega t}$
      II. $Z(y,t) = F(y+vt)$, where F is a function
      III. $X(z,t) = z^2 + c^2 t^2$
      IV. $Y(z,t) = 2.5z^2 - zt + 0.4t^2$

2. Conceptual [5 points]
   A long solenoid is placed inside a circuit as shown on the right. The current in the solenoid is ramped up linearly with time, resulting in a flux of $\phi = \alpha t$. The voltage over the two resistors (R1 and R2) in the circuit is measured between points A and B with ideal voltmeters (infinite resistance). Assume the wires have negligible resistance.
   Are V_1 and V_2 the same magnitude? Explain.

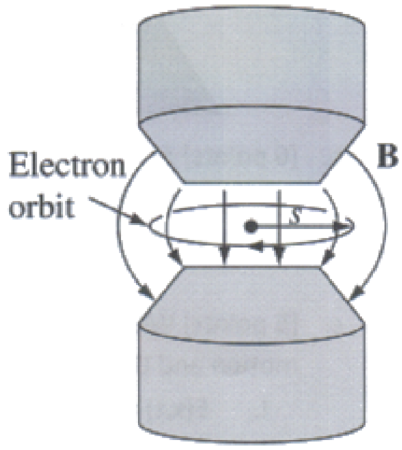


3. Betatron Accelerator [15 points]

In a Cyclotron particles are accelerated while circling around in a constant magnetic field. For these particles the centripetal acceleration is provided by the magnetic field (Lorentz force). This geometry results in a circular orbit with a constant angular velocity. In a different accelerator, the so called Betatron, electrons orbiting in a similar fashion, see figure below, can be accelerated (velocity increases) by increasing the magnetic field as a function of time. To keep the radius of the orbit constant during the acceleration process, the magnetic field also has to change as a function of radius. Thus B in a Betatron is a function of both radius and time and can be written as $B(r,t) = B_0(r)f(t)$. The requirement of a fixed radius during acceleration results in a fixed ratio between the average magnetic field inside the orbit with radius s, $B_{av}(s)$, and the local field at the radius of the particle orbit, B(s).

a. Which force does the work to accelerate the particle? How large is this force, expressed in the charge (*e*) and mass (m_e) of the electron, the radius of the orbit, *s*, and the magnetic field *B(r, t)*. [Hint: use the Maxwell equations in integral form.]

b. What is the ratio between the average magnetic field inside the orbit and the value of the magnetic field at the radius of the orbit? [Hint: Set the magnetic force equal to the centripetal force to find the relation between B and v, differentiate this relation with respect to time. Compare the resulting expression for the acceleration with the answer to part a)]

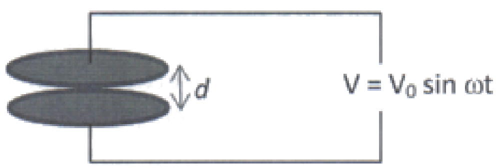


c. [2 pt Bonus] If the field at the particle radius S is ramped up according to $B(r,t) = B_{0S}(1+t)$ and $B_{0S} = 0.4$ T, what is then the energy increase per orbit?

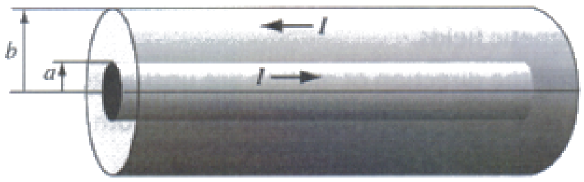
4. Charging Capacitor [10 points]

A capacitor consisting of two metal disks separated by a distance *d* is driven by an alternating voltage $V = V_0 \sin \omega t$.

a. Give an expression for the electric field assuming it is uniform. Is this a good assumption?
b. Determine the strength and direction of the magnetic field B



5. Energy and Inductance of a coaxial cable [15 points]

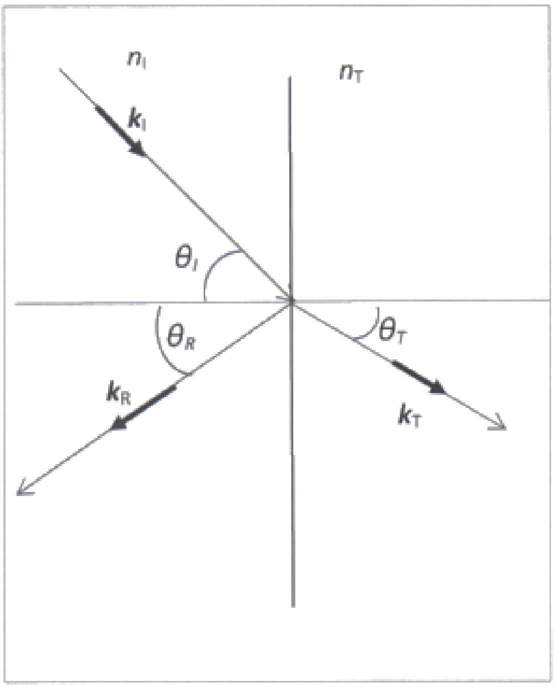


A long coaxial cable carries current *I*, see figure. Assume that the current only flows down the surface of the inner cylinder, radius *a*, and back along the outer cylinder, radius *b*.

a. Using Ampere's Law find the magnetic field between the cylinders.
b. Find the magnetic energy stored in a section of lenth *l*.
c. What is the self inductance *L* of the cable?

6. EM-waves in Matter (Snelius) [15 points]

A monochromatic wave $\boldsymbol{E}_i = \boldsymbol{E}_{0i} e^{i(\boldsymbol{k}_i \cdot \boldsymbol{r} - \omega_i t)}$ and $\boldsymbol{B}_i = \frac{1}{v_i}(\boldsymbol{k}_i \times \boldsymbol{E}_i)$ is incident on a material with index of refraction *n*, giving rise to a reflected wave and a transmitted wave. Assume that $\mu_I \approx \mu_T \approx \mu_0$.



a. [10 points] Use the electrodynamics boundary conditions and an appropriate expression for the reflected and transmitted waves to show that
   - the continuity condition on the frequency implies that $k_I = k_T \frac{n_I}{n_T}$ and $\theta_I = \theta_R$
   - the continuity equation of the electric field implies $\sin\theta_T / \sin\theta_I = n_I/n_T$,

   where the subscripts refer to the incident (I), reflected (R) and transmitted (T) wave respectively.
b. What is for each of the three waves the direction of the magnetic field and the Poynting vector. What does the Poynting vector represent?
c. [Bonus 3 points] Why is a metal surface highly reflective?

7. Fields of a moving point charge [15 points]

The fields of a moving charge can be found by repeated application of the Lorentz transformations for electric and magnetic fields.

The electric and magnetic fields along the direction of motion are unchanged. For the perpendicular direction the electric and magnetic fields can be found from

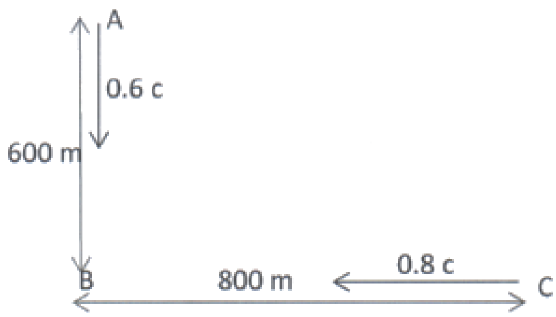
$$\boldsymbol{E}'_{\perp} = \gamma(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

$$\boldsymbol{B}'_{\perp} = \gamma(\boldsymbol{B}_{\perp} - \frac{1}{c^2}(\boldsymbol{v} \times \boldsymbol{E}))$$

Consider the following situation (see sketch below): Two charges, A and C, move towards an observer at B. Charge A moves in the vertical direction (down) with a velocity of 0.6 c. At t=0, A is at 600 m distance. C moves in the horizontal direction (left) with a velocity of 0.8 c. At t=0, C is at 800 m distance.

a. What is the relative velocity between A and C?
b. What are the electric and magnetic fields of A in the reference frame of C? Make a sketch of the fields showing all directions used in your answer. You need not provide a quantitative answer for the electric and magnetic fields, only relative size and directions.

[Hint: choose a smart coordinate system.]



Velocity addition:

$$u'_x = \frac{u_x - v}{(1 - \frac{u_x v}{c^2})}$$

$$u'_y = \frac{u_y}{\gamma(1 - \frac{u_x v}{c^2})}$$

$$u'_z = \frac{u_z}{\gamma(1 - \frac{u_x v}{c^2})}$$